accuracies, the solutions often become quite different because of sensitivity of the solutions due to the closeness of the planetary attraction. For higher accuracies, the curves quickly become nearly flat with little improvement in accuracy for large amounts of calculation time when using 14 digits of precision in the computations.

Comparing Curves A and B, for the finite-difference method of integration, the regularized equations of motion yield solutions with about 25% less calculation time than is required for the unregularized equations to attain the same accuracy. For the same calculation time, there is an increase in accuracy for the regularized solutions of about 10 to 20 times the unregularized solutions. Comparing Curves C and D, for the Runge-Kutta-Fehlberg method of integration, the regularized equations of motion yield solutions with about 35% to 45% less calculation time than is required for the unregularized equations to attain the same accuracy. For the same calculation time, there is an increase in accuracy for the regularized solutions of about 25 times the unregularized solutions.

A comparison between the finite-difference and the Runge-Kutta-Fehlberg methods of numerical integration for interplanetary trajectories is not intended here. The finite-difference method spends a significant amount of time in the Lagrangian starter for each step-size change and this time has been included in the calculation times of Fig. 1.

Conclusion

It appears that use of the regularized equations of motion yields solutions with greater accuracy while using less calculation time than unregularized equations, at least for the two methods of integration and for the particular trajectory used in the present study. The increased accuracy and less calculation time is due to the reduction in global truncation error and to the reduction in the number of integration steps. Due to the analytical result of Stiefel,[7] and the numerical results presented here and elsewhere,[9,12,13] it seems possible that the merits of the regularized equations of motion, as indicated in Fig. 1, may be extended to many other methods of integration and to the entire class of interplanetary trajectories with planetary close approaches.

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Fast Solution to Supersonic Plane Flat-Faced Blunt Body

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Nomenclature

c^* = sonic velocity
s, n = coordinate system
v_s, v_n = velocity components
v_{so} = body surface velocity (v_s at $n = 0$)
α = body incidence
δ = shock standoff distance, in n-direction for given s
χ = shock wave inclination relative to normal to freestream
γ = ratio of specific heats

HE first order, or one strip solution obtained by the method of integral relations,[1–3] has been found useful for accurately predicting inviscid high-speed flow about blunt bodies of arbitrary shape.[4–10] It has been found to give results indistinguishable from higher order solutions, and experiment, if freestream Mach number exceeds 8 (Ref. 1) and yields surprisingly good results in the neighborhood of the stagnation point at much lower Mach numbers.[3]

If, following Traugott,[6] we adopt body-oriented coordinates (s, n) using the stagnation point as origin (Fig. 1), the one strip method of integral relations reduces the transonic inviscid perfect gas flowfield equations to 3 simultaneous ordinary differential equations in the independent variable s for 3 dependent variables δ, χ, and v_{so}.

In the general case of a symmetric body of arbitrary shape, the problem can only be closed by applying a regularity condition through the sonic region singularity and iterating on the unknown

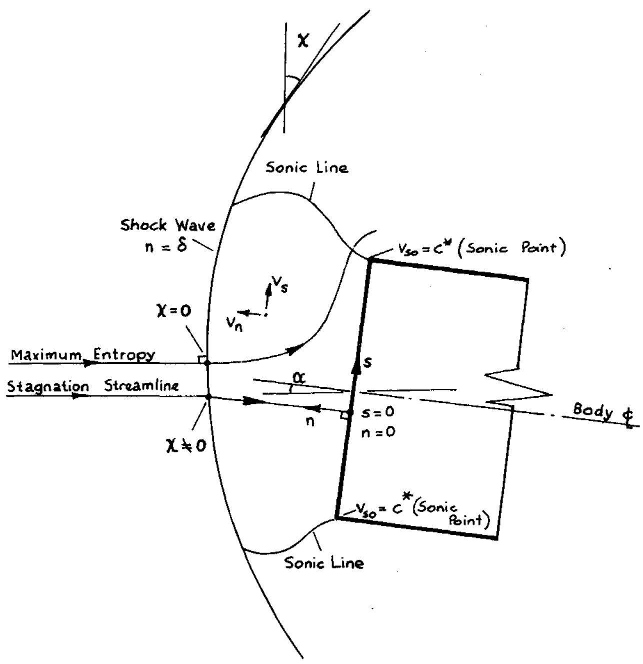

Fig. 1 Flowfield boundary and coordinate system.

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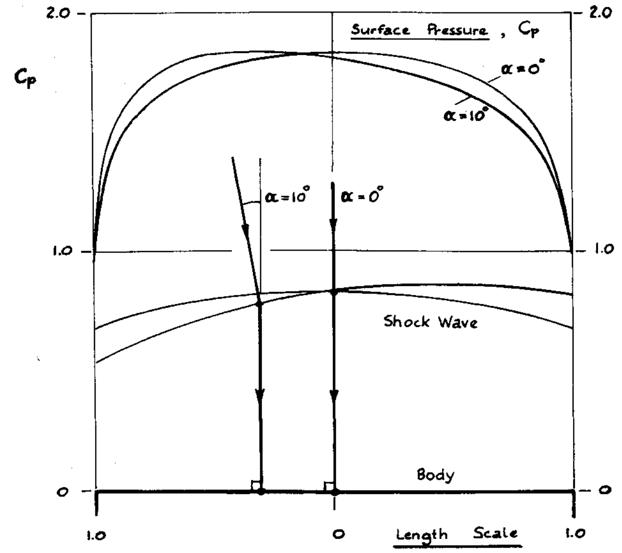

Fig. 2 Solution for supersonic flat plate; $M = 8.2$, $\gamma = 1.4$.

initial value of shock standoff distance. For plane bodies at incidence, the position of the stagnation point is also unknown, but can nominally be found by using the regularity conditions at both upper and lower sonic points. However, the problem is less complicated in a one strip analysis. Brong and Leigh,[11] have shown that for this case the stagnation streamline is a straight line normal to the body surface. Thus, for a given stagnation point the slope of the stagnation streamline and hence of the tangent to the shock wave can be found by simple plane shock theory. For a flat-faced blunt body (Fig. 1), therefore, both of these angles are known initially. Now, the regularity condition at the sonic point is in general controlled by an expression of the form

$$dv_{so}/ds = N/D$$

where D contains $c^{*2} - v_{so}^2$ and must be zero at a sonic point, and where N must be finite at a sharp corner, and zero at a smooth corner. Thus, this determines the basis upon which iteration is performed if it is necessary to search for the stagnation point shock standoff distance.

However, for a sharp cornered flat-faced plane body at arbitrary incidence, similarity requires that the body shape and size have no influence on the solution. Consequently, a solution can be obtained immediately for any assumed or unit shock standoff distance, by integrating from the stagnation point found by the method of Brong and Leigh until the sonic condition is satisfied. Although this can be termed an indirect rather than direct solution, since the body is found for a given initial shock, the direct solution for a given body size can in fact be found by simple scaling of lengths in the nominally indirect answer. Using about 250 integration steps on each side of the body, computing time on a CDC 6600 is about 10 sec. Some results are shown in Fig. 2.

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Bending of an Orthotropic Cantilever

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THE shear stress distribution at composite beam interfaces is a problem of continuing concern to materials design engineers. In order to develop insight into the important parameters involved in evaluating such materials, improved analytical techniques are necessary for studying experimental results and providing useful design guidelines.

In a previous paper,[1] the plane stress solution applicable to a thin-walled cantilever beam with end load has been discussed. The present paper extends the preceding result to include the influence of beam width for the Saint Venant solution to the bending of a composite orthotropic beam, Fig. 1.

Using a semi-inverse technique we consider the Saint Venant solution for bending of a composite orthotropic cantilever beam Fig. 1 subject to the elastic field equations and the following stress and strain conditions[2]:

$$\begin{aligned} &\tau_{xy} = \sigma_x = \sigma_y = 0 \\ &\tau_{xz} = \tau_{xz}(x, y), \quad \tau_{yz} = \tau_{yz}(x, y), \quad \sigma_z = E_z \varepsilon_z \\ &\varepsilon_z = c_2 zx + c_3 x + c_5 z + c_6 \end{aligned} \tag{1}$$

The strain displacement equations, the equation of equilibrium and the substitution

$$\bar{w}(x, y) = Dx^3 + Fxy^2 + \alpha x^2 + \beta y^2 + w(x, y) \tag{2}$$

yield the desired equations

$$\begin{aligned} \bar{w} = {} & [-E_z/G_{zx} + v_{zx}](c_2/6)x^3 + v_{zy}(c_2/2)x^2 y + \\ & [-E_z/G_{xz} + v_{zx}](c_5/2)x^2 + v_{zy} c_5 y^2/2 + w(x, y) \end{aligned} \tag{3a}$$

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